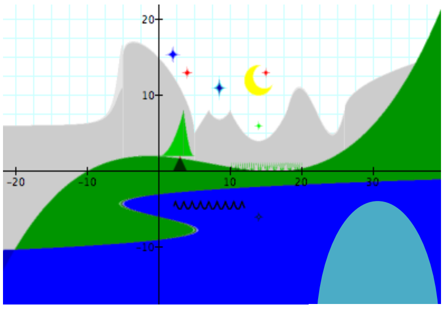
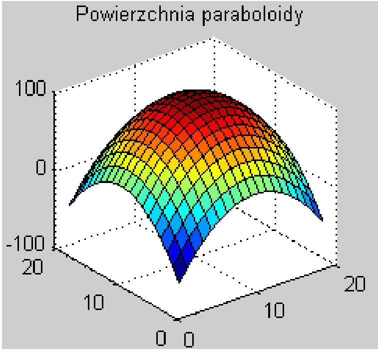
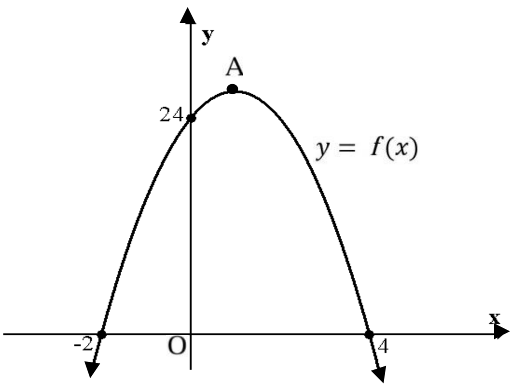
Algebraic Functions





To excel in this topic, a thorough grasp of the following is essential:

- The 4 algebraic functions, viz. the straight line $y = mx + c$, the parabola $y = ax^2 + bx + c$, the exponential function $y = a.b^x + q$ and the hyperbola $y = \frac{k}{x-p} + q$. The cubic function $y = ax^3 + bx^2 + cx + d$ will be treated under the topic *Calculus*.
- The *shape* of each of the above and their inverses.
- The influence of the parameters, a, b, c, p, q, etc. on each of the above functions.
- Where applicable, their x-and y-intercepts, vertices and asymptotes.
- The influence of shifts, vertical and/or horizontal and also reflections in the x-axis, the y-axis and in the $y = x$ and $y = -x$ lines on the *equations* and *graphs* of these five functions.



1.1 The two roots -2 and 4 $\Rightarrow y = a(x+2)(x-4)$
and the point $(0; 24)$ $\Rightarrow 24 = a(0+2)(0-4)$
$\therefore -8a = 24 \quad \therefore a = -3$
$\therefore y = -3(x+2)(x-4)$

1.2 $y = -3(x+2)(x-4) = -3(x^2 - 2x - 8)$

$= -3[x^2 - 2x + (\ldots)^2 - (\ldots)^2 - 8]$

$= -3[x^2 - 2x + (-1)^2 - (-1)^2 - 8]$

$= -3[(x-1)^2 - 9]$

$= -3(x-1)^2 + 27$

1.3 A$(1; 27)$

1.4 For f to have two equal roots, the graph must be shifted down vertically to touch the x-axis, i.e. the x-axis must be a tangent to the graph at its turning point. $\therefore$ Graph must be shifted vertically downwards by 27 units.

1.5 For reflection in the y-axis, change the sign of x.
$\therefore p(x) = f(-x) = -3([-x]-1)^2 + 27 = -3(x^2 + 2x + 1) + 27 = -3x^2 - 6x + 24$

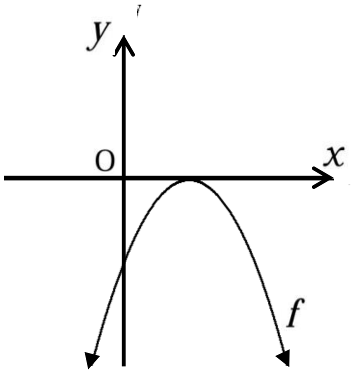
OR... from the standard form of the eq. of $f(x)$, viz. $y = -3x^2 + 6x + 24$, we get
$p(x) = f(-x) = -3(-x)^2 + 6(-x) + 24$ i.e. $p(x) = -3x^2 - 6x + 24$.

2.1 <u>The graph of $f(x) = y = ax^2 + bx + c$</u>

$a < 0$ and $b > 0 \Rightarrow x = -\frac{b}{2a} = -\left(\frac{+}{-}\right) = +$

$\therefore$ axis of symmetry to the ***right*** of y-axis and 'arms' facing down (maximum vertex)

$b^2 - 4ac = 0 \;\Rightarrow$ equal roots

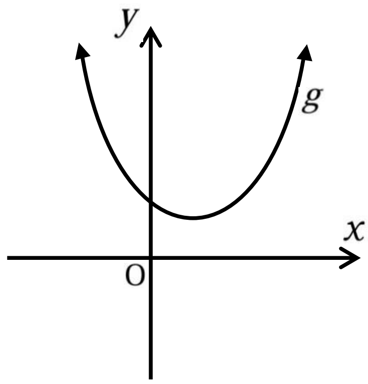


2.2 <u>The graph of $g(x) = y = ax^2 + bx + c$</u>

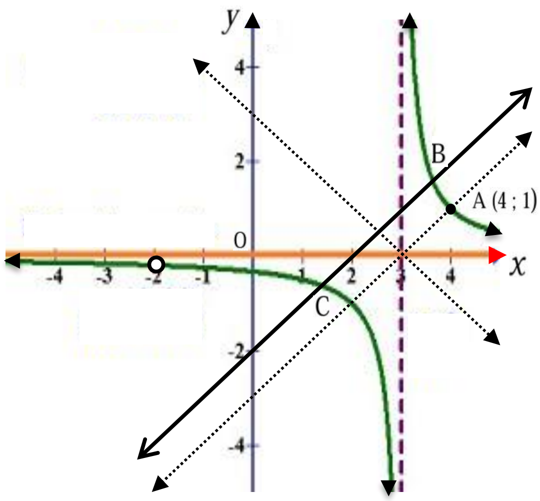
$a > 0$ and $b < 0 \Rightarrow x = -\frac{b}{2a} = -\left(\frac{-}{+}\right) = +$

$\therefore$ axis of symmetry to the ***right*** of y-axis and 'arms' facing up (or minimum vertex)

$b^2 - 4ac < 0 \;\Rightarrow$ non real roots



3.1 With respect to $f(x) = \frac{a}{x-p} + q,\; x \neq -2$ we see from the asymptotes of f that $p = 3$ and $q = 0$ and substituting A$(4; 1)$ in the eq. we have: $1 = \frac{a}{4-3} + 0 \;\; \therefore a = 1$.
With respect to $g(x) = mx + c$, it is quite clear that from the x- and y-intercepts of g, $m = 1$ and $c = -2$.



3.2 Domain of f: $(-\infty; \infty),\; x \neq -2\,;\; x \neq 3$

Equation of f: $y = \frac{1}{x-3}$, $x \neq -2$.

3.3 Solving simultaneously by substitution,

where $y = x - 2 \;\Rightarrow x - 2 = \frac{1}{x-3} \;\Rightarrow\; (x-2)(x-3) = 1 \;\Rightarrow x^2 - 5x + 5 = 0$

$x = \frac{5 \pm \sqrt{5}}{2} \;\; \therefore\; x \approx 1{,}38$ or $x \approx 3{,}62 \;\therefore$ C$(1{,}38; -0{,}62)$ and B$(3{,}62; 1{,}62)$.

3.4 The equations of the lines of symmetry are $y = x + c$ and $y = -x + c$ respectively. Using the point of intersection of the two asymptotes viz. $(3; 0)$, gives the values of c. $\therefore\; 0 = 3 + c$ $\therefore c = -3$ and eq. is $y = x - 3$ and $0 = -3 + c \;\; \therefore c = 3$ and eq. $y = -x + 3$.
(**<u>Note</u>**: The axes of symmetry of hyperbolae <u>*always*</u> pass through the point of intersection of the two asymptotes…namely $(3; 0)$ in this case, and although not asked here, we have drawn in these two (as dashed) lines for identification and clarification purposes only).

3.5.1 Where f is above g $\;\therefore (-\infty; 1{,}38]$

3.5.2 Where $(+)(-) \leq 0$ ***<u>or</u>*** $(-)(+) \leq 0 \Rightarrow f$ and g on opposite sides of the x-axis $\;\therefore\; [2; 3)$

3.5.3 Where (x values)x(g values) $< 0 \;\; \therefore (0; 2)$

4.1 Asymptote: $y = -8$

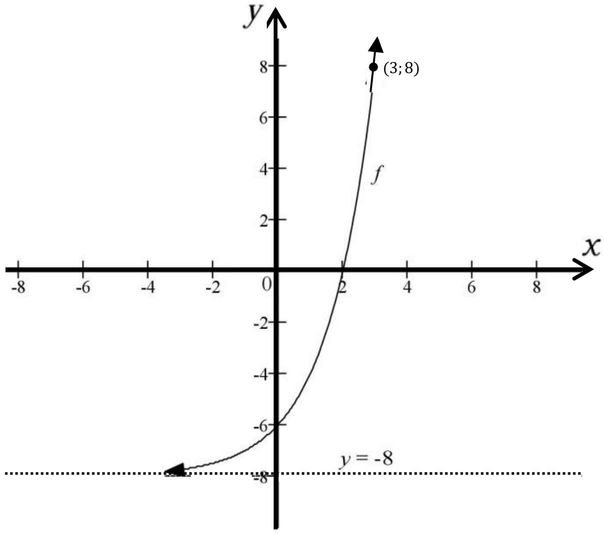
y-intercept: Let $x = 0$ $\therefore$ $y = 2^{0+1} - 8$

$= -6$

x-intercept: Let $y = 0$ $\therefore 0 = 2^{x+1} - 8$

$2^{x+1} = 8 \Rightarrow 2^{x+1} = 2^3 \therefore x = 2$

For an accurate sketch, plot a 3rd point, say $x = 3$ $\therefore y = 2^{3+1} - 8 = 16 - 8 = 8$



4.2 Reflection in the y-axis, change the sign of x. $\therefore g(x) = f(-x)$

$= 2^{-x+1} - 8$ **or** $g(x) = 2.\left(\frac{1}{2}\right)^x - 8$

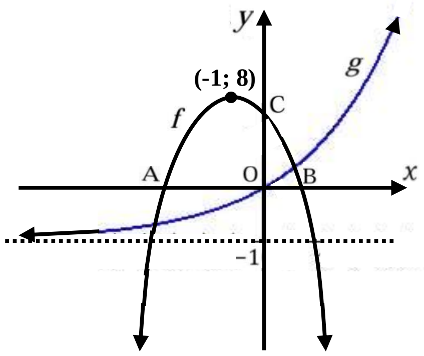
4.3 $\mathrm{Q}(0; -6)$

5.1 We have two points for the graph of g, viz. (0; 0) and (1; 2). $\therefore 0 = d^0 + q$ $\therefore$ $q = -1$.
B$(1; 2)$ gives: $2 = d^1 - 1$ $\therefore$ $d = 3$.

5.2 c = 6 ...(y-intercept) A$(-3; 0)$ gives: $0 = a(-3)^2 + b(-3) + 6$ $\therefore$ $9a - 3b = -6$ ....❶
B$(1; 0)$ gives: $0 = a(1)^2 + b(1) + 6$ $\therefore$ $a + b = -6$ $\therefore$ $3a + 3b = -18$ ....❷
① + ②: $12a = -24$ $\therefore$ $a = -2$ and from ① or ② $b = -4$

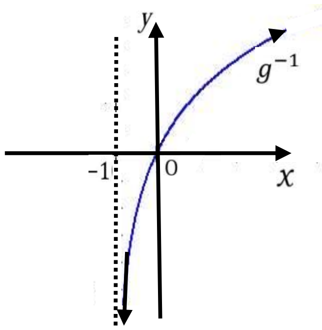
5.3 $f'(x).g(x) < 0 \Rightarrow m_f \text{x}\, y_g < 0 \Rightarrow m_f$ and y_g to have opposite signs and therefore require x-value of the turning point (vertex) of f, viz. $x = -1$ (halfway between the two roots) OR $x = -\frac{b}{2a} = -1$

$(-\infty; -1) \cup (0; \infty)$

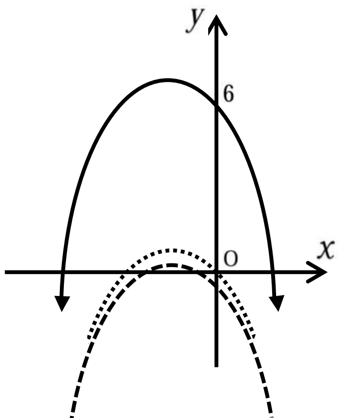


5.4 Equation of g: $y = 3^x - 1$ $\therefore$ Inverse of g:
$x = 3^y - 1$ i.e. $x + 1 = 3^y$ $\therefore$ $y = \log_3(x + 1)$

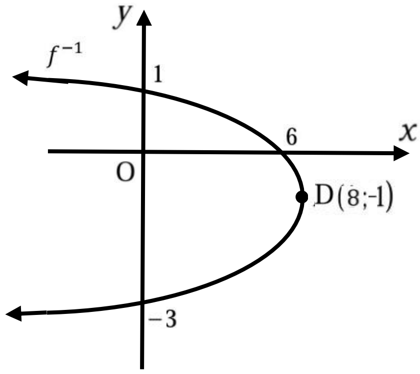
5.5 Domain of g^{-1}. <u>1st method</u>: Algebraic approach.
As per definition, log N exists only if N > 0
$\therefore$ $x + 1 > 0 \Rightarrow x > -1,\ x \in R$
<u>2nd method</u>: Graphic approach. See sketch of g^{-1} (to the right) for an easy and visual verification.



5.6 $k > -6$
(Explanation...
-6 is the maximum negative value that can be added to the equation of f, giving it a y-intercept through the origin... the fine broken line, for it to have two roots of opposite sign. If the value added is less than -6, i.e. $-8 \leq k < -6$, the equation will have two negative roots... the coarse broken line as shown in the diagram. It is obvious that for $k < -8$, the equation will have non real roots).



5.7 (i) Turning point of f: $x = -\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$
and $y = -2(-1)^2 - 4(-1) + 6 = 8$
$\therefore$ Turning point of f: $(-1; 8)$
Let D be the turning point of f^{-1} $\therefore$ D$(8; -1)$
(ii) Eq. of f^{-1} : 1st write eq. of f in its t.pt form
$\therefore y = -2(x+1)^2 + 8$ $\therefore$ Eq. of f^{-1} becomes
$x = -2(y+1)^2 + 8 \Rightarrow x - 8 = -2(y+1)^2$
$\therefore y + 1 = \pm\sqrt{\frac{8-x}{2}}$ $\therefore$ $f^{-1}(x) = y = -1 \pm \sqrt{\frac{8-x}{2}}$



(**Note**: In order to determine the equation of the inverse of a quadratic function that's equation contains a ' *bx* ' term, the equation must first be written in the $y = a(x-p)^2 + q$ form before attempting to change the *x*'s and *y*'s around).

6.1 No. Only functions that have a one-to-one correspondence will have inverses that are also functions. The inverse of a more-to-one correspondence is not a function.
g has a one-to-one correspondence, $\therefore$ g^{-1} is a function. (It passes the vertical line test).
f has a more-to-one correspondence, $\therefore$ f^{-1} is not a function. (It fails the vertical line test).

6.2 The domain can be changed in two ways: i) Restricting it to $(-\infty; -1]$ will result in an inverse whose graph is shown in 6.3 (fig. 1). ii) Restricting the domain to $[-1; \infty)$ will result in an inverse whose graph is shown in 6.3 (fig. 2).

6.3 Fig. 1

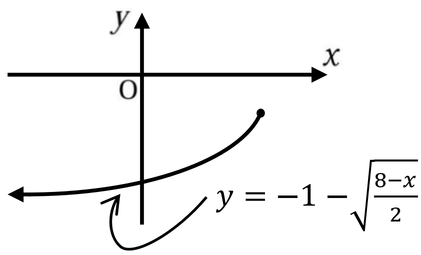
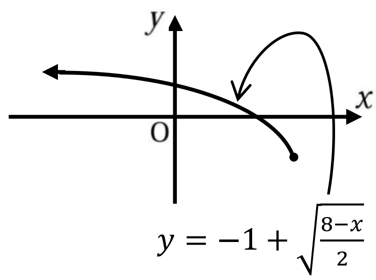


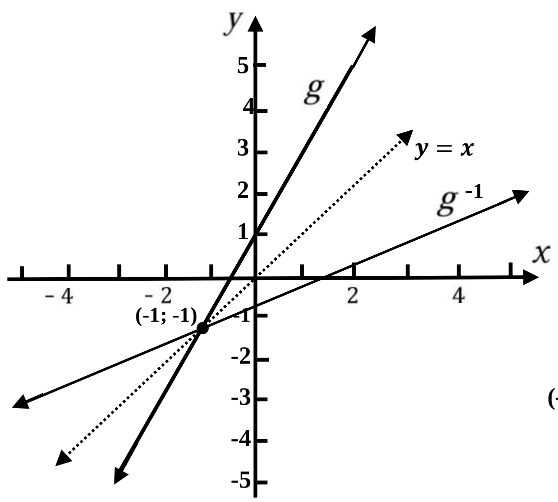
Fig. 2



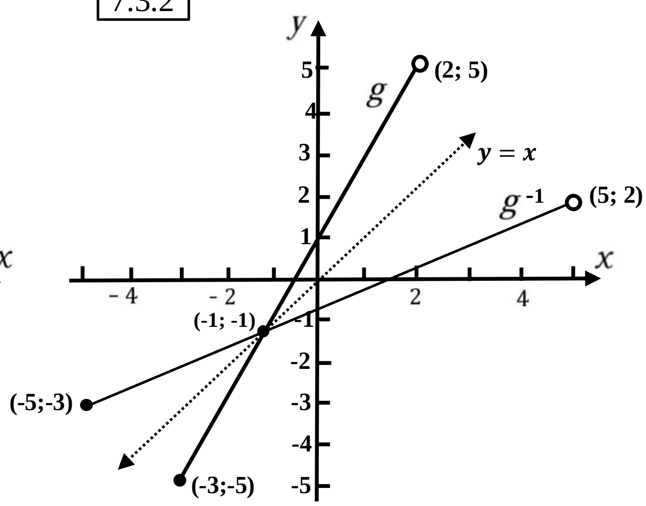
7.1 Inverse of g: $x = 2y + 1$ $\therefore y = 0{,}5x - 0{,}5$ $\therefore$ eq: $y = 0{,}5x - 0{,}5$, $x \in [-5; 5)$
(**Note**: The range values of g's domain now becomes the domain of g^{-1})

7.2 $2x + 1 = 0{,}5x - 0{,}5 \Rightarrow 4x + 2 = x - 1$ $\therefore x = -1$ $\therefore (x; y) = (-1; -1)$.

7.3.1



7.3.2



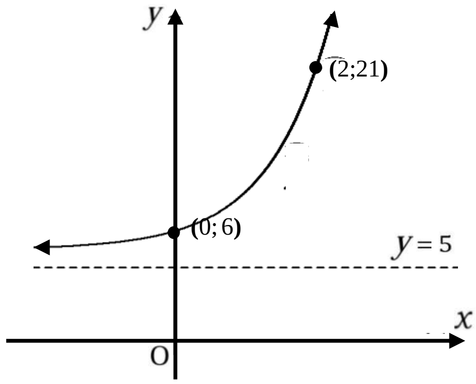
8.1 Substituting (6;0) $\Rightarrow\ 0=\log_b(6+q)\ \therefore\ 6+q=b^0$
$\therefore\ q=-5$. Subst. $(21;2)$ we have $2=\log_b(21-5)$
$\Rightarrow 16=b^2\ \therefore\ b=4\ \therefore\ y=f(x)=\log_4(x-5)$

8.2 $f(x)\le -2 \Rightarrow \log_4(x-5)\le -2\ \therefore\ x-5\le 4^{-2}$
$\therefore\ x\le 5\frac{1}{16}$. But domain of f is $(5;\infty)$
$\therefore\ x=(5;5\frac{1}{16}]$

8.3 Eq. of f^{-1} is inverse of $f\ \therefore\ x=\log_4(y-5)$
$\therefore\ y-5=4^x\ \therefore\ y=4^x+5=f^{-1}(x)$.

8.4 Graph of $f^{-1}(x)=4^x+5$ drawn on the right.

Graph of $f^{-1}(x)=4^x+5$

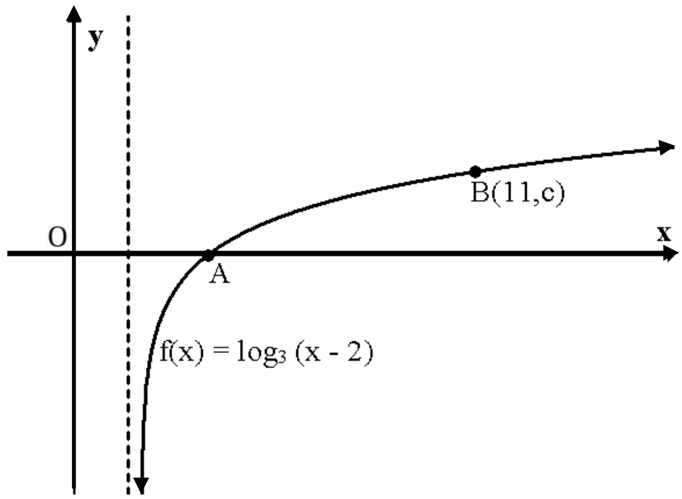


9.1 Substituting the point $(11\,;\,c)$ in eq.
of f viz. $y=\log_3(x-2)$
$\Rightarrow c=\log_3(11-2)=\log 9\ \therefore 9=3^c$
$\therefore\ c=2$

9.2 $x=2$

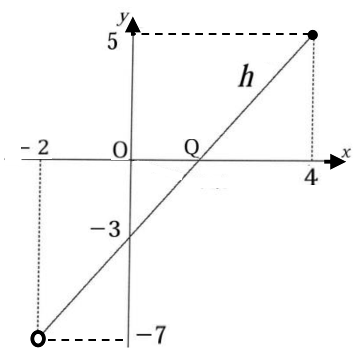
9.3 $f(x)<2 \Rightarrow \log_3(x-2)<2$
$\therefore\ x-2<3^2\ \therefore\ x<11$
But domain of f is $(2;\infty)$
$\therefore\ x\in(2;11)$

9.4 Reflection in the x-axis
$\Rightarrow$ change the sign of y.
$\therefore\ -y=\log_3(x-2)\ \therefore\ y=-\log_3(x-2)$

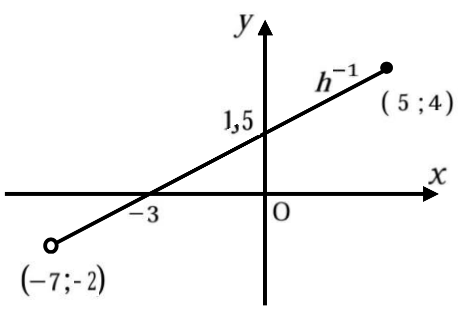


10.1 For x-intercept, let $y=0\ \therefore\ 0=2x-3\ \therefore\ x=1{,}5$
$\therefore$ Q(1,5 ; 0)

10.2 For equation of h^{-1}: $x=2y-3\ \therefore\ y=1{,}5x+1{,}5$
Since h^{-1} is the inverse of h, the domain of h^{-1}
is the range of h. (sketch given here for clarification)
$\therefore$ Domain of h^{-1} : $-7<x\le 5,\ x\in R$ **OR (-7; 5]**
$\therefore\ h^{-1}(x)=1{,}5x+1{,}5,\quad -7<x\le 5,\ x\in R$



10.3 The graph of h^{-1} is sketched below.

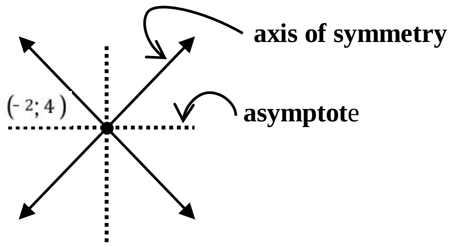


10.4 For x value(s) where $h(x) = h^{-1}(x)$: Set $2x - 3 = 1{,}5x + 1{,}5$ $\therefore$ $0{,}5x = 4{,}5$ $\therefore x = 9$

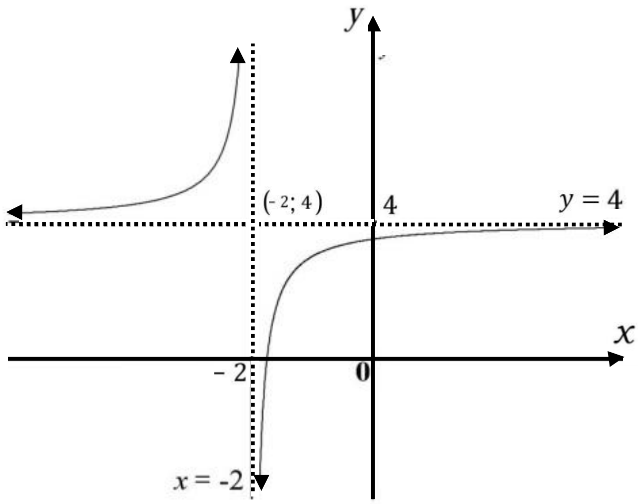
10.5.1 $h(x) = f'(x)$ $\Rightarrow$ that f is a quadratic function with equation $f(x) = x^2 - 3x + k$, and since the coefficient of x^2 is positive $(a > 0)$, the function has a minimum value at its turning point $x = -\frac{b}{2a} = -\left(\frac{-3}{2(1)}\right)$ = 1,5, and which falls within the domain given.

10.5.2 $h(x) = 2x - 3$ represents the gradient of any tangent to the graph of f. Since 4 is the maximum positive value of the domain of f, $f'(4)$ will be the maximum gradient of the tangent. $\therefore$ $f'(4) = 2(4) - 3 = 5$ i.e. maximum gradient is 5.

11.1 Information given tells us:
i) one asymptote at $x = -2$
ii) The other asymptote intersects the axis of symmetry, viz. $y = x + 6$ and substituting $x = -2$ in equation $\Rightarrow y = x + 6 = -2 + 6 = 4$ $\therefore y = 4$.
iii) An increasing function implies that the branches are in 2nd and 4th quadrants.



(**<u>Note</u>**: As we have no further information, the x- and y-intercepts cannot accurately be determined).



11.2.1 Reflection in the x-axis, change the sign of y $\therefore$ $-y = \frac{-a}{x+2} + 4$

$\therefore$ $y = \frac{-a}{x+2} - 4$

11.2.2 Reflection in the y-axis, change the sign of x. $\therefore$ $y = \frac{-a}{-x+2} - 4$

$\therefore$ $y = \frac{a}{x-2} - 4$

11.2.3 Reflection in the $y = x$ line, change x's and y's around. $\therefore$ $x = \frac{-a}{y+2} + 4$ $\therefore$ $x - 4 = \frac{-a}{y+2}$

$\therefore$ $y + 2 = \frac{-a}{x-4}$ $\therefore$ $y = \frac{-a}{x-4} - 2$.

12.1 Given that $(8; 4)$ lies on g $\Rightarrow$ $8 = \sqrt{4a}$ $\therefore 16 = 4a$ $\therefore a = 4$

12.2 $g(x)$ is defined for , $x \in R$ **OR** $[\mathbf{0}; \infty)$

12.3 Range of g: $y \geq 0,\ y \in R$ **OR** $[\mathbf{0}; \infty)$

12.4 Equation of g^{-1}: $x = \sqrt{4y}$ $\therefore$ $x^2 = 4y$ $\therefore$ $y = \frac{x^2}{4}$, $x \geq 0$

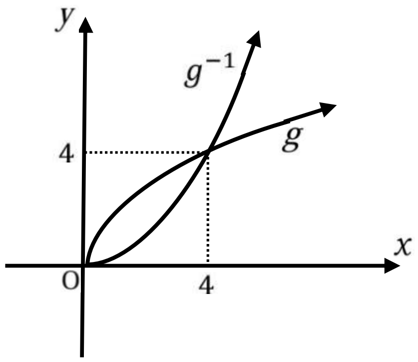
12.5 Graphs shown on the right.

$g(x) = g^{-1}(x) \Rightarrow \sqrt{4x} = \frac{x^2}{4}$

$\therefore 4x = \frac{x^4}{16} \Rightarrow 64x = x^4 \therefore x^4 - 64x = 0$

$x(x^3 - 64) = 0 \therefore x = 0$ or $x^3 = 64$

$\therefore x = 0$ or $x = 4$



12.6.1 Intersection(s) of g and $h \Rightarrow \sqrt{4x} = x - 4$

$\therefore 4x = (x-4)^2 \Rightarrow 4x = x^2 - 8x + 16$

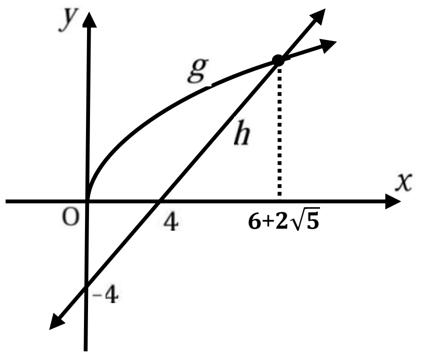
$\therefore x^2 - 12x + 16 = 0$

$\therefore x = 6 + 2\sqrt{5}$ or $x = 6 - 2\sqrt{5}$. But $x \neq 6 - 2\sqrt{5}$ ….(an extraneous root as a result of squaring both sides of eq. earlier on). This can be verified by drawing the graphs of g and h, where it can be clearly seen there is only one point of intersection, viz, $x = 6 + 2\sqrt{5}$

$\therefore x = 6 + 2\sqrt{5}$ is the only solution.

12.6.2 $g(x) > h(x) \Rightarrow$ values of x where the graph of g is above the graph of h.

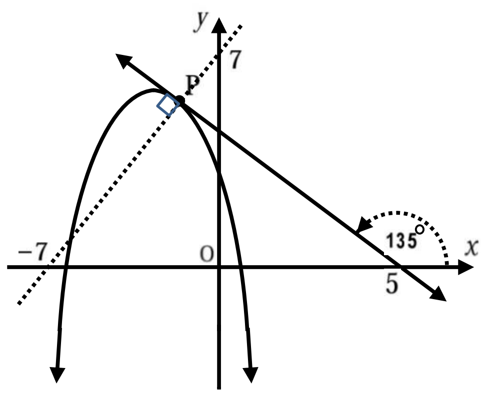
$\therefore 0 < x < 6 + 2\sqrt{5}$.



13.1 Turning point of f: $x = -\frac{b}{2a} = -\frac{-5}{2(-2)} = -1\frac{1}{4}$

and $y = -2(-1{,}25)^2 - 5(-1{,}25) + 3 = 6\frac{1}{8}$

$\therefore$ coordinates of turning pt. of f: $\left(-1\frac{1}{4}\,;6\frac{1}{8}\right)$



13.2 $m_g = \tan 135^0 = -1$ and $m_{tangent} = f'(x)$

$\Rightarrow f'(x) = -4x - 5 = -1 \therefore x = -1$

$\Rightarrow y = -2(-1)^2 - 5(-1) + 3 = 6$

$\therefore$ P$(-1; 6)$

13.3 Substitute $(-1; 6)$ into eq. of g with $m_g = -1 = a$

$\therefore g(x) = y = ax + q \Rightarrow 6 = -1(-1) + q$

$\therefore q = 5 \therefore$ Equation of g: $y = -x + 5$

13.4 A normal is $\perp$ is to the tangent $\therefore m_{normal} = 1$ and eq. of normal: $y = x + c$, and as it also passes through P$(-1; 6) \Rightarrow 6 = 1(-1) + c \therefore c = 7$ and eq. of normal: $y = x + 7$ with x-intercept at N$(-7; 0)$ and from eq. of g, its x-intercept is Q$(5; 0)$

$\therefore$ NQ $= 12$ units.

13.5 $d > 5,\ d \in R$ **OR** $(5; \infty)$

14.1 (i) For roots of f let $y = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad or \quad x = -1$

(ii) Turning point of f:

$x = \frac{-2}{2(-1)} = 1$

$y = -(1)^2 + 2(1) + 3 = 4$

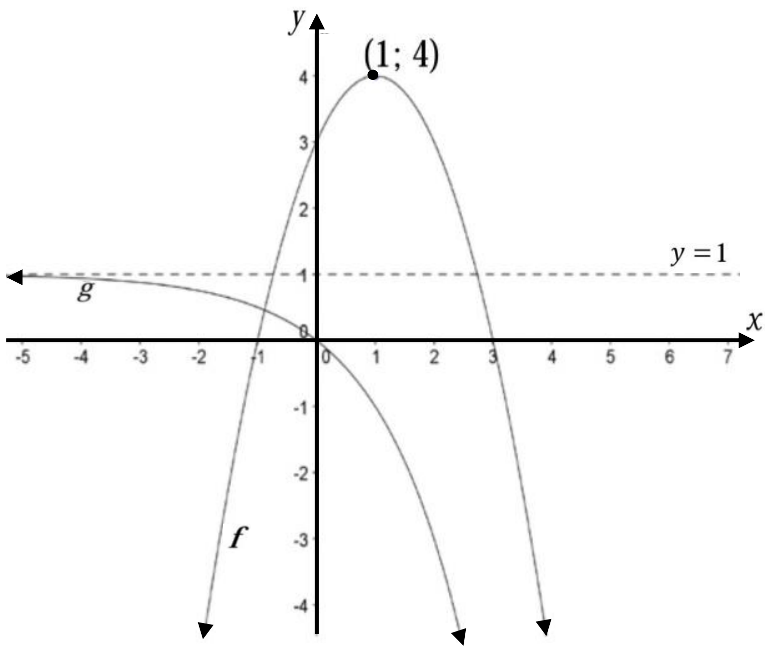
$\therefore (1; 4)$

(iii) x- and y-intercepts of g:

$0 = 1 - 2^x \qquad y = 1 - 2^0$

$2^x = 2^0 \qquad y = 0$

$x = 0$



14.2 Average gradient $= \frac{f(0) - f(-3)}{0 - (-3)}$

$= \frac{3 - (-12)}{3}$

$= 5$

14.3 $-1 \leq x \leq 0$ or $x \geq 3$

14.4 For h to be a tangent to the x-axis, f must be translated 4 units down. $\therefore$ c = -4

14.5 $-g(x)$ is the reflection of g in the x-axis $\Rightarrow$ graph will again pass through the origin, and $+ 1 \Rightarrow$ y-intercept is one unit higher. $\therefore (0; 1)$ is the y-intercept of $-g(x) + 1$.

14.6 Reflection in the y-axis $\Rightarrow$ change sign of x: $\therefore k(x) = 1 - 2^{-x}$ or $k(x) = 1 - \left(\frac{1}{2}\right)^x$.

15.1 Asymptotes: $x = 3$ and $y = -1$.

15.2 Domain: $x \in R,\ x \neq 3$

15.3 $d = m_g = \tan 76^0 = 4{,}0107 \ldots \approx 4$. And for e, subst. point B$(3; 6)$ into $y = 4x + e$ $\therefore 6 = 4(3) + e \quad \therefore e = -6$.

15.4 For coordinates of A and C, solve $f(x) = g(x) \Rightarrow \frac{2}{x-3} - 1 = 4x - 6 \ \therefore \frac{2}{x-3} = 4x - 5$ $\therefore 2 = (4x - 5)(x - 3) \ \therefore 4x^2 - 17x + 13 = 0 \ \therefore (4x - 13)(x - 1) = 0 \ \therefore x = 1$ or $x = \frac{13}{4}$ $\therefore$ A$(1; -2)$ and C$(3\frac{1}{4}; 7)$

15.5 $[1; 3) \cup \left[\frac{13}{4}; \infty\right)$ **OR** $1 \leq x < 3$ or $x \geq \frac{13}{4},\ x \in R$

15.6 Substitute $(3; -1)$ into $y = x + c \Rightarrow -1 = 3 + c \quad \therefore$ c $= -4 \ \therefore$ Eq: $y = x - 4$.

15.7 $x = -1$ and $y = 3$.

15.8 Setting $y = 0$ in eq. of $f \Rightarrow$ D$(5; 0) \Rightarrow f$ must shift 5 units to the left. $\therefore$ Eq. of h: $y = \frac{2}{(x+5)-3} - 1 \ \therefore h(x) = \frac{2}{x+2} - 1$

16.1 a) C
b) D
c) D
d) A, C and D.

16.2 a) B
b) B
c) C

17. (**Note**: Very often, the equation of an hyperbola is not given in its (normal) standard form, but in the form as given above. It is therefore important to master the technique of manipulating the equation algebaically so that it is written in the standard form, and from which (form) the asymptotes can clearly be discerned. This is achieved by splitting up the numerator into a number of terms, each term with its own denominator, in such a way that all the x's in the numerator are cancelled out. Follow this carefully, also in the next question, question no. 18).

17.1 $f(x) = \frac{x-5}{x+2} = \frac{x+2-7}{x+2} = \frac{x+2}{x+2} + \frac{-7}{x+2} = 1 + \frac{-7}{x+2} \quad \therefore f(x) = \frac{-7}{x+2} + 1$

17.2 Eq. of asymptotes of f: $x = -2$ and $y = 1$.

17.3 Domain of f^{-1}: $x \in R,\ x \neq 1$ **OR** $(-\infty; 1) \cup (1; \infty)$.

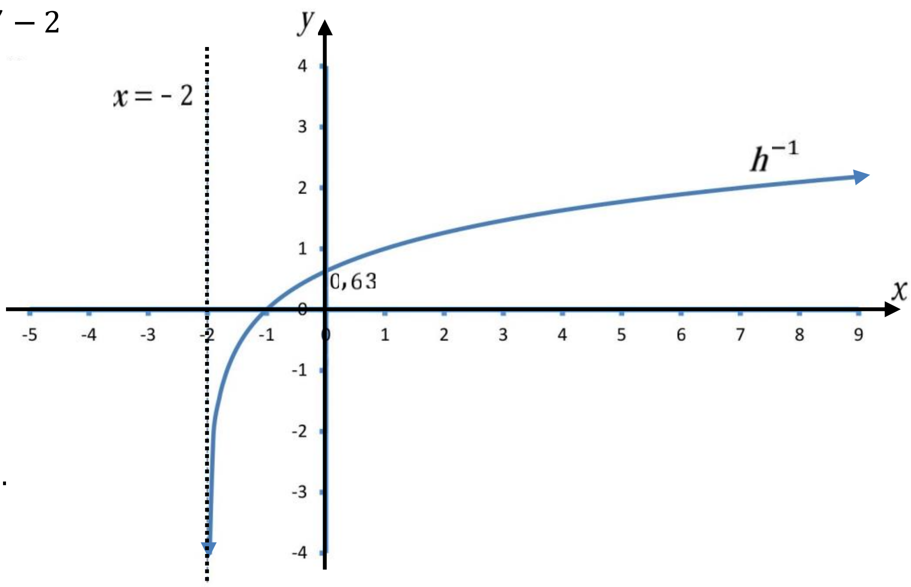
18.1 $f(x) = \frac{2x+1}{x-3} = \frac{x+x+1}{x-3} = \frac{x-3+x-3+7}{x-3} = \frac{x-3}{x-3} + \frac{x-3}{x-3} + \frac{7}{x-3} = 1 + 1 + \frac{7}{x-3} = 2 + \frac{7}{x-3}$

$\therefore f(x) = \frac{7}{x-3} + 2$

18.2 Eq. of asymptotes of f: $x = 3$ and $y = 2$.

18.3 Domain of f^{-1}: $x \in R,\ x \neq 2$ **OR** $(-\infty; 2) \cup (2; \infty)$.

19.1 Eq. of inverse: $x = 3^y - 2$
$\therefore x + 2 = 3^y$
$\therefore\ y = \log_3(x+2)$



19.2 Graph showing all the important aspects, e.g. y-intercept:
$y = \log_3(0+2)$
$y = \log_3 2 = 0{,}6309$ ..

19.3 $x > -1,\ x \in R$ **OR** $(-1; \infty)$

19.4 To find the new equation of h, we need to determine the point of intersection of the asymptotes of the hyperbola, $g(x)$, and to do that, we write its equation in standard form:

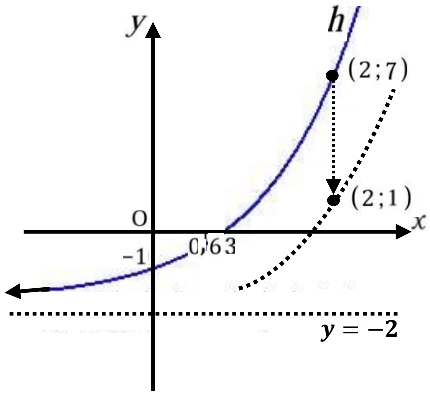
$g(x) = \frac{x-6}{x-2} = \frac{x-2-4}{x-2} = \frac{x-2}{x-2} + \frac{-4}{x-2}$

$\therefore\ g(x) = \frac{-4}{x-2} + 1$

$\therefore$ point of intersection of asymptotes is $(2; 1)$ which means h has to be translated vertically downwards to pass through $y = 1$. To determine by how many units, we must find h's y-value at $x = 2$ before the translation. $\therefore$ $y = 3^2 - 2 = 7$

$\Rightarrow$ graph is shifted 6 units downwards….

(from $y = 7$ to $y = 1$)



$\therefore$ new eq. of $h(x)$: $y = 3^x - 2 - 6$ i.e. $y = 3^x - 8$.

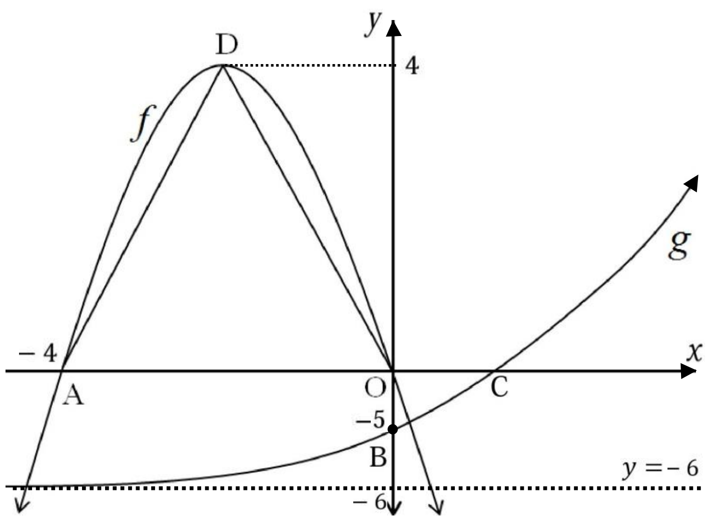
20.1 (i) For roots of f, set $y = 0$ $\therefore 0 = -x^2 - 4x$ $\therefore x(x+4) = 0$ $\therefore$ $x = 0$ or $x = -4$

(ii) For y-intercept of g, set $x = 0$ $\therefore y = 2^0 - 6 = -5$ $\therefore$ B$(0; -5)$

(iii) T.pt. D: $x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = -2$ and $y = -(-2)^2 - 4(-2) = 4$ $\therefore$ D$(-2; 4)$

Area of ΔAOD $= \frac{1}{2}$basexheight.

$= \frac{1}{2}(4)(4) = 8$ square units



20.2 From the diagram it is clear that $f \cap g$ (i.e the intersection of f and g) to to have one negative root and the other equal to zero, the graph of f must be shifted to the left to intersect the graph of g at B, and for two negative roots, it must be shifted further to the left just passed $\mathrm{B}(0;-5)$. Now, $-(x-p)^2-4(x-p)=2^x-6$ gives one neg. root and one zero root at $x=0$. $\therefore$ $-(0-p)^2-4(0-p)=2^0-6$ $\Rightarrow -p^2+4p=1-6=-5$
$\therefore p^2-4p+5=0$ $\therefore (\mathrm{p}-1)(\mathrm{p}-5)=0$ $\therefore p=1$ or $p=5$. But $x \neq 5$ … n/a here.
$\Rightarrow f$ to shift just more than one unit to the left $\Rightarrow x-p>x+1$ $\therefore -\mathrm{p}>1$ $\therefore \mathrm{p}<-1$.

20.3 Again, from the diagram, it can be seen that if the graph of g is shifted vertically upwards to pass through the origin, ***a shift of five units***, $f(x)=g(x)$ will have one negative root and one root equal to zero, making $k=-6+5=-1$, i.e. $g(x)=2^x-1$
$\therefore$ For two roots of opposite signs, g must be shifted > 5 units. $\therefore$ $k<-1$.

oooooooooooooO **END OF ANSWERS ON ALGEBRAIC FUNCTIONS** Oooooooooooooo

<u>Space for some notes</u>